Scalar Spectrum from a Dynamical Gravity/Gauge model.

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We show that a Dynamical AdS/QCD model is able to reproduce the linear Regge trajectories for the light-flavor sector of mesons with high spin and also for the scalar and pseudoscalar ones. In addition the model has confinement by the Wilson loop criteria and a mass gap. We also calculate the decay amplitude of scalars into two pion in good agreement to the available experimental data.

Keywords: AdS/QCD; Confinement; Regge Trajectory.

1. Introduction

Over the years experiments confirm that strong interaction is successfully described by Quantum Chromodynamics (QCD). For very high energies one can calculate physical amplitudes analytically using the QCD Lagrangian due to asymptotic freedom. On the other hand we have a lack of analytical tools to analyze the low energy sector. Important properties of the infrared physics of the strong interaction such as confinement, mass gap and linear Regge trajectories remains unexplained by QCD.

In 1974 't Hooft proposed a duality between the large N (number of colors) limit of QCD and string theory [1]. This represented the first dual representation of a gauge theory by a string model. In 1998 Maldacena[2] proposed a mapping between operators in Conformal Field Theory (CFT) and fields of a $N = 4$ Type IIB string field theory in a ten-dimensional space-time $AdS_5 \times S_5$. The most interesting fact of this duality is that the strong-coupling regime of large-N_c gauge theories can be approximated (in low-curvature regions) by weakly coupled and hence analytically treatable classical gravities. The drawback is that CFT is not QCD. Consequently, the $N = 4$ Type IIB string field in $AdS_5 \times S_5$ does not have many important properties of strong interactions as confinement and a mass gap.

A direct way for searching for a QCD dual is introducing D-branes in the theory. They are responsible for breaking in part supersymmetry and for the introduction of flavor. For example, the addition of N_f D7 probe branes ($D3 - D7$ model[3]) can

be interpreted as the introduction of flavor in the AdS/CFT. In the supergravity side it is a four-dimensional $N = 2$ supersymmetric large-N gauge theory. Although a Type II B $N = 2$ has a running coupling constant, it does not has confinement. There is a vast literature addressing these topics and for a review see [4]. In all those models (top-down) we obtain a one-dimensional differential equation in holographic coordinate to calculate the mass spectra and they do not lead to a Regge spectrum for meson excitations (see e.g. [5]). This fact suggested an other way of searching for the corresponding QCD dual. We propose an effective 5d action that can reproduce basic properties of strong interaction and we explore the phenomenological aspects of this model in a bottom-up approach.

The first model with this idea was proposed by Polchinski and Strassler[6]. This model (hard-wall) is a slice of AdS with an IR boundary condition that introduces the QCD scale. It implements the counting rules which govern the scaling behavior of hard QCD scattering amplitudes by the conformal invariance of AdS_5 in the UV limit. In spite of reproducing a large amount of hadron phenomenology[7] it does not have linear Regge trajectories. A soft-wall model[8] was created to correct this problem, where the AdS_5 geometry is kept intact while an additional dilaton background field is introduced. This dilaton soft-wall model indeed generates linear Regge trajectories $m^2_{n,S} \sim n + S$ for light-flavor mesons of spin S and radial excitation level n. (Regge behavior can alternatively be encoded via IR deformations of the AdS_5 metric [9,10].) However, the resulting vacuum expectation value (vev) of the Wilson loop in the dilaton soft wall model does not exhibit the area-law behavior in contrast to a linearly confining static quark-antiquark potential. It happens because the model uses an AdS metric which is not of a confining type by the Wilson loop analysis [11,12]. In addition the soft-wall model background is not a solution of a dual gravity. Csaki and Reece [13] analyzed the solutions of a 5d dilaton-gravity Einstein equations (see also[14]) using the superpotential formalism. They concluded that it would not be possible to solve those equations and obtain a linear confining background without introducing new ingredients. They suggested to analyze a tachyon-dilaton-graviton model, and this idea was successfully implemented in [15].

We took an alternative route and we show[16] that we can obtain a linear confining background as solution of the dilaton-gravity coupled equations. Within our proposal of a self-consistent dilaton-gravity model, the mass spectrum of the high spin mesons stays close to a linear Regge trajectory for the lower excitations, where experimental data exists, while an exact linear behavior is approached for high spin and mass excitations.(Using similar approach in [17] is proposed a different AdS deformation in order to reproduce the QCD running coupling.)

1.1. *Hadronic Resonances in Dynamical AdS/QCD model*

The action for five-dimensional gravity coupled to a dilaton field is:

$$S = \frac{1}{2k^2}\int d^5x\sqrt{g}\left(-R - V(\Phi) + \frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi\right), \tag{1}$$

where k is the Newton constant in 5 dimensions and $V(\Phi)$ is the scalar field potential. We will be restricted to the metric family: $g_{MN} = e^{-2A(z)}\eta_{MN}$, where η_{MN} is the Minkowski one. Minimizing the action, we obtain a coupled set of Einstein equations for which the solutions satisfy the following relations:

$$\Phi' = \sqrt{3A'^2 + 3A''}\ ,\ V(\Phi) = \frac{3e^{2A}}{2}\left(A'' - 3A'^2\right). \tag{2}$$

The 5d action for a gauge field $\phi_{M_1\ldots M_S}$ of spin S in the background is given by[8]

$$I = \frac{1}{2}\int d^5x\sqrt{g}e^{-\Phi}\left(\nabla_N\phi_{M_1\ldots M_S}\nabla^N\phi^{M_1\ldots M_S}\right). \tag{3}$$

As in [8] and [18], we utilize the axial gauge. To this end, we introduce new spin fields $\widetilde{\phi}_{\ldots} = e^{2(S-1)A}\phi_{\ldots}$. We also make the substitution $\tilde{\phi_n} = e^{B/2}\psi_n$ and obtain a Sturm-Liouville equation

$$\left(-\partial_z^2 + \mathcal{V}_{eff}(z)\right)\psi_n = m_n^2\psi_n, \tag{4}$$

where $B = A(2S-1) + \Phi$ and $\mathcal{V}_{eff}(z) = \frac{B'^2(z)}{4} - \frac{B''(z)}{2}$. Hence, for each metric A and dilaton field Φ consistent with the solutions of the Einstein equations, we obtain a mass spectrum m_n^2. Due to the gauge/gravity duality this mass spectrum corresponds to the mesonic resonances in the 4d space-time.

Now we will focus on scalar mesons (also analyzed in [19,20]). The action[21]

$$I = \frac{1}{2}\int d^4x dz\sqrt{|g|}\left(g^{\mu\nu}\partial_\mu\varphi(x,z)\partial_\nu\varphi(x,z) - \frac{M_5^2}{\Lambda_{QCD}^2}\varphi^2\right), \tag{5}$$

describes a scalar mode propagating in the dilaton-gravity background. Factorizing the holographic coordinate dependence as $\varphi(x,z) = e^{iP_\mu x^\mu}\varphi(z)$ with $P_\mu P^\mu = m^2$, and redefining the string amplitude as $\psi_n(z) = \varphi_n(z) \times e^{-(3A+\Phi)/2}$, we have a Sturm-Liouville equation

$$\left[-\partial_z^2 + \mathcal{V}(z)\right]\psi_n = m_n^2\psi_n, \tag{6}$$

where the string-mode potential is $\mathcal{V}(z) = \frac{B'^2(z)}{4} - \frac{B''(z)}{2} + \frac{M_5^2}{\Lambda_{QCD}^2}e^{-2A(z)}$ with $B = 3A+\Phi$. (Note that $B = (2S-1)A+\Phi$ for the spin nonzero states [8].) The AdS/CFT correspondence states that the wave function should behave as z^τ, where $\tau = \Delta - \sigma$ (conformal dimension minus spin) is the twist dimension for the corresponding interpolating operator that creates a given quark-gluon configuration [6]. The five-dimensional mass chosen as [22] $M_5^2 = \tau(\tau - 4)$, fixes the UV limit of the dual string amplitude with the twist dimension.

2. Phenomenological Results

Our aim was to construct a metric ansatz that is AdS in the UV and allows for confinement, mass gap and Regge trajectories by tailoring its IR behavior. With

these constraints we found the metric:

$$A(z) = Log(\xi z \Lambda_{QCD}) + \frac{(\xi z \Lambda_{QCD})^2}{1 + e^{(1-\xi z \Lambda_{QCD})}} \tag{7}$$

For scalars $\xi = 0.58$. To distinguish the pion states in our model, the fifth dimensional mass was rescaled according to $M_5^2 \to M_5^2 + \lambda z^2$ (see [10]). The model is constrained by the pion mass, the slope of the Regge trajectory and the twist 2 from the operator $\bar{q}\gamma^5 q$. The results for the Regge trajectories for f_0 and pion are shown in figure 1. The pion modes are calculated with ξ =0.88 and $\lambda = -2.19\text{GeV}^2$. For high spin mesons, see figure 2, we have an equation to obtain the scale factor $\xi = S^{-0.3329}$ in order to keep the slope of the Regge trajectories fixed. (Note that in our previous work[16] we adopted a slightly different ansatz.)

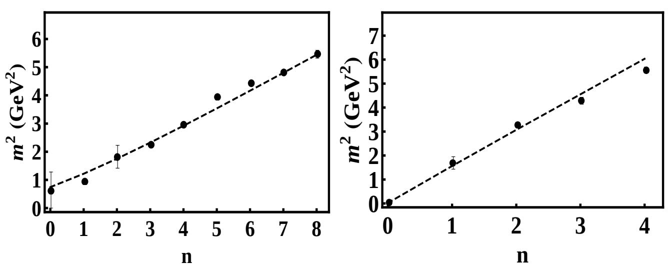

Fig. 1. Regge trajectory for f_0 (left panel) and pion (right panel) from the Dynamical AdS/QCD model with $\Lambda_{QCD} = 0.3$ GeV. Experimental data from PDG.

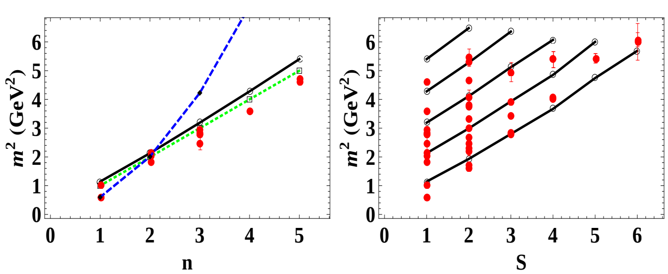

Fig. 2. Radial excitations of the rho meson in the hard-wall (dashed line), soft-wall [8] (dotted line) and our dynamical soft-wall (solid line, for $\Lambda_{QCD} = 0.3$ GeV) backgrounds (left panel). Dynamical AdS/QCD spectrum for spin excitations (right panel). Experimental data from PDG.

3. Decay Amplitudes

The f_0's partial decay width into $\pi\pi$ are calculated from the overlap integral (h_n) of the normalized string amplitudes (Sturm-Liouville form) in the holographic coordinate dual to the scalars (ψ_n) and pion (ψ_π) states,

$$h_n = k \int_0^\infty dz\ \psi_\pi^2(z)\psi_n(z)\ , \quad \text{with} \quad \int_0^\infty dz \psi_m(z)\psi_n(z) = \delta_{mn}\ . \tag{8}$$

The constant k has dimension $\sqrt{mass}$ fitted to the experimental value of the $f_0(1500) \to \pi\pi$ partial decay width. The Sturm-Liouville amplitudes of the scalar (pseudoscalar) modes are normalized just as a bound state wave function in quantum mechanics [23,24], which also corresponds to a normalization of the string amplitude. The overlap integral for the decay amplitude, h_n, is the dual representation of the transition amplitude $S \to PP$ and therefore the decay width is given by $\Gamma^n_{\pi\pi} = \frac{1}{8\pi}|h_n|^2 \frac{p_\pi}{m_n^2}$, where p_π is the pion momentum in the meson rest frame.

The known two-pion partial decay width for the f_0's given in the particle listing of PDG[25], are calculated with Eq. (8) and shown in Table I. The width of $f_0(1500)$ is used as normalization. In particular for $f_0(600)$ the model gives a width of about 500 MeV, while its mass is 860 MeV. The range of experimental values quoted in PDG for the sigma mass and width are quite large as depicted in Table I. The analysis of the E791 experiment gives $m_\sigma = 478^{+24}_{-23} \pm 17$ MeV and $\Gamma_\sigma = 324^{+42}_{-40} \pm 21$ MeV [26]. The width seems consistent with our model while the experimental mass appears somewhat smaller. The CLEO collaboration [27] quotes $m_\sigma = 513 \pm 32$ MeV and $\Gamma_\sigma = 335 \pm 67$ MeV, and a recent analysis of the sigma pole in the $\pi\pi$ scattering amplitude from ref.[28] gives $m_\sigma = 441^{+16}_{-8}$ MeV and $\Gamma_\sigma = 544^{+18}_{-25}$ MeV. Other analysis of the σ-pole in the $\pi\pi \to \pi\pi$ scattering amplitude present in the decay of heavy mesons indicates a mass around 500 MeV [29].

Table 1. Two-pion decay width and masses for the f_0 family.Experimental values from PDG. †Mixing angle of 20^o. *Fitted value.

Meson	M_{exp}(GeV)	M_{th}(GeV)	$\Gamma^{exp}_{\pi\pi}$(MeV)	$\Gamma^{th}_{\pi\pi}$(MeV)
$f_0(600)$	0.4 - 1.2	0.86	600 - 1000	535
$f_0(980)$	0.98± 0.01	1.10	∼15-80	$42^\dagger$
$f_0(1370)$	1.2 - 1.5	1.32	∼41-141	141
$f_0(1500)$	1.505±0.006	1.52	38±3	38*
$f_0(1710)$	1.720±0.006	1.70	∼ 0-6	5
$f_0(2020)$	1.992±0.016	1.88	—	0.0
$f_0(2100)$	2.103±0.008	2.04	—	1.2
$f_0(2200)$	2.189±0.013	2.19	—	2.5
$f_0(2330)$	2.29-2.35	2.33	—	2.8

4. Conclusions

In this work we obtain a spectrum of high spin mesons, scalar and pseudoscalars in the light-flavor sector, in agreement to experimental data available using a Dynamical AdS/QCD model. In addition we calculate the decay amplitude of scalar mesons into two pions. We introduce a mixing angle for $f_0(980)$ of $\pm 20^o$, that corresponds to a composite nature by mixing, e.g., $s\bar{s}$ with light non-strange quarks[30]. An absolute value of the mixing angle between $\sim 12^\circ$ to 28° fits $\Gamma^{\pi\pi}$ within the experimental range. Currently we are including the strange meson sector[31] in the Dynamical model. As a future challenge we also want to introduce finite temperature and calculate the meson spectrum, as done for glueballs within the soft- and hard-wall models [32]. Finally it could be compared to a large N analysis at finite temperature using lattice simulations recently delivered by Panero[33].

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